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Profit Analysis of Two Unit Cold Standby System with Two Types of Failure under Inspection Policy and Discrete Distribution

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Abstract - In this paper the two identical unit cold standby systems with single repairman has been discussed.The concept of inspection policy has also been introduced for detecting the kind of failures (major or minor) before the failed unit get repaired by some repair mechanism.The model has been design for the system to calculate the various important measures of reliability i.e MTSF, steady state availability, busy period of repairman and inspector, profit function using discrete distribution and regenerative point techniques. Profit function and MTSF are also analyzed graphically.

Keywords: Regenerating point technique**,** MTSF, Availability, Inspection, Busy period and Profit function

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1. INTRODUCTION

Reliability is essential for proper utilization and maintenance of any system and equipments.Therefore it had gained much importance among manufacturers. Reliability deals with the development of new techniques for increasing the system effectiveness by reducing the frequency of failures and minimizing the high maintenance costs.A large literature exist in the area of reliability theory of standby systems.Many researchers had analyzed reliability models with failure and repair time by using continuous distribution.Aggarwal (2010) had considered the cold standby system with two types of failure using exponential distribution.Said, Salah, Sherbeny (2005) had analyzed the profit function for two unit cold standby system with preventive maintenance and random change in units.In this paper they considered the concept of inspection that is being performed after the failed unit get repaired by repairman to check the satisfactory result from repairman.Haggag (2009) had analyzed reliability models where the observed data were found to be large, for which the continous distribution was considered to be an accurate distribution.But it's not always true as sometimes we come across some situations when the observed values are small. In such cases, continuous distribution might not adequately describe a discrete random variable. Then one has to deal reliability models with discrete distribution to obtain the various reliability measures of the system effectiveness such as the MTSF, availability and busy period of repairman etc.

In the field of reliability using discrete distribution Bhardwaj (2009) had analyzed two unit redundant systems with imperfect switching and connection time. In his research he had also analysed two identical unit standby and parallel systems with two types of failure.Gupta (2007) had studied two identical unit parallel systems with Geometric failure and repair time distributions. Now in this paper the two identical coldstandby system was anzlyzed by introducing the concept of inspection policy for detecting the two types of failure where inspection and repair time are taken as geometric distribution. Initially one unit is operative and other is in cold standby. On the failure of a unit, an inspection is being done first to investigate the one out of two types (minor or major) of failures. This helps the repairman to repair an exact failure of the failed unit.Preference will be given to the minor failure on the major one. The repairman time taken by minor is less as compaired to major.

The model is analysed stochastically and the expressions for the various reliability measures of system effectiveness such as mean time to system failure, steady state availability, and busy period for both inspector and repairman were obtained.Graphs were also been drawn to analysed the behavior of MTSF and profit function with respect to repair and failure rate.

2. MODEL DESCRIPTION

The following assumptions are associated with the model:

- A system consists of two identical cold standby units arranged in a parallel network. Initially one unit is operative and other unit is in cold standby.
- Upon the failure of an operative unit, the cold standby unit becomes operative instantaneously.
- The system is assumed to be in the failed state when both units together were in failed conditions whether the cause of failure is major or minor.
- A single repairman is available to repair both types of failed unit after being inspected.

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Preference will be given to the minor failure on the major one.

- A repaired unit's works as good as new.

3 NOMENCLATURE

O	:	Unit is in operative mode
S	:	Unit is in standby mode
a	:	Probability that unit goes to failed state with major failure.
b	:	Probability that unit goes to failed state with minor failure.
F_i	:	Unit is in failure mode and under inspection.
F_{Mr} / F_{Mw}	:	Unit is in major failure mode and under repair /waiting for repair.
F_{mr} / F_{mw}	:	Unit is in minor failure mode and under repair /waiting for repair.
p_1 / q_1	:	Probability of the failed unit inspected satisfactory or not.
p_2	:	Probability of the failure.
r	:	Unit is under repair.
$q_{ij}(t)$ / $Q_{ij}(t)$	:	p.d.f and c.d.f of first passage time from regenerative state i to regenerative state j.
$P_{ij}(t)$	:	Steady state transition probability from state S_i to S_j.
μ_i	:	Mean sojourn time in state S_i.

Up States

S_0 = (O, S),
S_1 = (F_i, O),
S_2 = (F_{Mr}, O),
S_3 = (F_{mr}, O)

Down State

S_4 = (F_{Mr}, F_i),
S_5 = (F_{mr}, F_i),
S_6 = (F_{Mr}, F_{Mw}),
S_7 = (F_{Mw}, F_{mr}),
S_8 = (F_{mr}, F_{Mw}),
S_9 = (F_{mr}, F_{mw}).

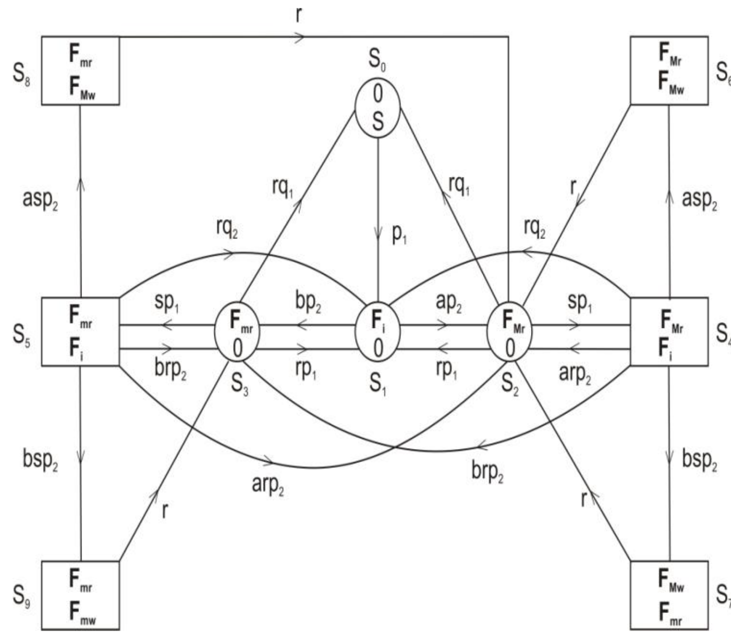

Figure-1: Transition Diagram

4 TRANSITION PROBABILITIES AND SOJOURN TIMES

$$Q_{01}(t) = \frac{p_1[1-q_1^{(t+1)}]}{1-q_1} \qquad Q_{12}(t) = \frac{ap_2[1-q_2^{(t+1)}]}{1-q_2}$$

$$Q_{13}(t) = \frac{bp_2[1-q_2^{(t+1)}]}{1-q_2}$$

$$Q_{20}(t) = Q_{30}(t) = \frac{rq_1[1-(q_1 s)^{t+1}]}{1-q_1 s}$$

$$Q_{21}(t) = Q_{31}(t) = \frac{rp_1[1-(q_1 s)^{t+1}]}{1-q_1 s}$$

$$Q_{24}(t) = Q_{35}(t) = \frac{sp_1[1-(q_1 s)^{t+1}]}{1-q_1 s}$$

$$Q_{41}(t) = Q_{51}(t) = \frac{rq_2[1-(q_2 s)^{t+1}]}{1-q_2 s}$$

$$Q_{42}(t) = Q_{52}(t) = \frac{arp_2[1-(q_2 s)^{t+1}]}{1-q_2 s}$$

$$Q_{43}(t) = Q_{53}(t) = \frac{brp_2[1-(q_2 s)^{t+1}]}{1-q_2 s}$$

$$Q_{46}(t) = Q_{58}(t) = \frac{asp_2[1-(q_2 s)^{t+1}]}{1-q_2 s}$$

$$Q_{47}(t) = Q_{59}(t) = \frac{bsp_2[1-(q_2 s)^{t+1}]}{1-q_2 s}$$

$Q_{62}(t) = Q_{72}(t) = Q_{82}(t) = Q_{93}(t) = \dfrac{r[1-s^{(t+1)}]}{1-s}$ (1-12)

The steady state transition probabilities from state S_i to S_j can be obtained from

$$P_{ij} = \lim_{t\to\infty} Q_{ij}$$

It can be verified that

$P_{01} = 1,\quad P_{12} + P_{13} = 1,$
$P_{20} + P_{21} + P_{24} = 1,$
$P_{30} + P_{31} + P_{35} = 1,$
$P_{41} + P_{42} + P_{43} + P_{46} + P_{47} = 1,$
$P_{51} + P_{52} + P_{53} + P_{58} + P_{59} = 1$
$P_{62} = P_{72} = P_{82} = P_{93} = 1$

(13-19)

5 Mean Sojourn Times

Let T_i be the sojourn time in state S_i (i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9), then mean sojourn time in state S_i is given by

$$\mu_i = E(T_i) = \sum_{t=0}^{\infty} P(T_i > t)$$

so that

$$\mu_0 = \frac{1}{1-q_1},\quad \mu_1 = \frac{1}{1-q_2},\quad \mu_2 = \mu_3 = \frac{1}{1-q_1 s},$$

$$\mu_4 = \mu_5 = \frac{1}{1-q_2 s},\quad \mu_6 = \mu_7 = \mu_8 = \mu_9 = \frac{1}{1-s}$$

(20-24)

Mean sojourn time (m_{ij}) of the system in state S_i when the system is to transit into S_j is given by

$$m_{ij} = \sum_{t=0}^{\infty} t\, q_{ij}(t)$$

$m_{01} = q_1\mu_0,\qquad m_{12} + m_{13} = q_2\mu_1,$
$m_{20} + m_{21} + m_{24} = m_{30} + m_{31} + m_{35} = sq_1\, \mu_2,$
$m_{41} + m_{42} + m_{43} + m_{46} + m_{47} = m_{51} + m_{52} + m_{53} + m_{58} + m_{59} = sq_2\, \mu_4$
$m_{62} = m_{72} = m_{82} = m_{93} = s\, \mu_6$

(25-29)

6 Reliability and Mean Time to System Failure

Let $R_i(t)$ be the probability that system works satisfactorily for atleast t epochs 'cycles' when it is initially started from operative regenerative state S_i (i = 0, 1, 2, 3).

$R_0(t) = Z_0(t) + q_{01}(t-1) © R_1(t-1)$
$R_1(t) = Z_1(t) + q_{12}(t-1) © R_2(t-1) + q_{13}(t-1) © R_3(t-1)$
$R_2(t) = Z_2(t) + q_{20}(t-1) © R_0(t-1) + q_{21}(t-1) © R_1(t-1)$
$R_3(t) = Z_3(t) + q_{30}(t-1) © R_0(t-1) + q_{31}(t-1) © R_1(t-1)$

(30-33)

Taking geometric transformation on both sides, we get

$$R_0(h) = \frac{N_1(h)}{D_1(h)}$$

The mean time to system failure is

$$\mu_i = \lim_{h\to 1} \frac{N_1(h)}{D_1(h)} - 1 = \frac{N_1}{D_1}$$

(34)

where

$N_1 = (\mu_0 - 1)(1 - P_{21}) + \mu_1 + \mu_2 + P_{20} = \mu_0(1 - P_{21}) + \mu_1 + \mu_2 - P_{24}$
$D_1 = 1 - P_{20} - P_{21} = P_{24}$

(35-36)

7 Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch t when it is initially started from regenerative state S_i by simple probabilistic argument the following recurrence relations are obtained.

$A_0(t) = Z_0(t) + q_{01}(t-1) © A_1(t-1)$
$A_1(t) = Z_1(t) + q_{12}(t-1) © A_2(t-1) + q_{13}(t-1) © A_3(t-1)$
$A_2(t) = Z_2(t) + q_{20}(t-1) © A_0(t-1) + q_{21}(t-1) © A_1(t-1) + q_{24}(t-1) © A_4(t-1)$
$A_3(t) = Z_3(t) + q_{30}(t-1) © A_0(t-1) + q_{31}(t-1) © A_1(t-1) + q_{35}(t-1) © A_5(t-1)$
$A_4(t) = q_{41}(t-1) © A_1(t-1) + q_{42}(t-1) © A_2(t-1) + q_{43}(t-1) © A_3(t-1) + q_{46}(t-1) © A_6(t-1) + q_{47}(t-1) © A_7(t-1)$
$A_5(t) = q_{51}(t-1) © A_1(t-1) + q_{52}(t-1) © A_2(t-1) + q_{53}(t-1) © A_3(t-1) + q_{58}(t-1) © A_8(t-1) + q_{59}(t-1) © A_9(t-1)$
$A_6(t) = q_{62}(t-1) © A_2(t-1)$
$A_7(t) = q_{72}(t-1) © A_2(t-1)$
$A_8(t) = q_{82}(t-1) © A_2(t-1)$
$A_9(t) = q_{93}(t-1) © A_3(t-1)$

(37-46)

By taking geometric transformation and solving the equation

$$A_0(h) = \frac{N_2(h)}{D_2(h)}$$

and

$Z_0(t) = q_1^t,\qquad Z_1(t) = q_2^t,\qquad Z_2(t) = Z_3(t) = (q_1 s)^t,$
$Z_4(t) = Z_5(t) = (q_2 s)^t \quad Z_6(t) = Z_7(t) = Z_8(t) = Z_9(t) = s^t$

(47)

Hence,

$$Z_i(h) = \mu_i$$

The steady state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t)$$

Hence, by applying 'L' Hospital Rule, we get

$$A_0 = -\frac{N_2(1)}{D'_2(1)} \quad (48)$$

where

$N_2(1) = \mu_0 P_{20}(1 - P_{24}P_{47}) + \mu_1\{1 - P_{24} + P_{24}P_{41}\}(1 - P_{24}P_{47}) + \mu_2(1 - P_{24}P_{47})$

$D'_2(1) = -\{q_1\mu_0 P_{20}(1 - P_{24}P_{47}) + q_2\mu_1[1 - P_{24} + P_{24}P_{41}](1 - P_{24}P_{47}) + sq_1\mu_2(1 - P_{24}P_{47}) + sq_2\mu_4 P_{24}(1 - P_{24}P_{47}) + s\mu_6 P_{24}[P_{46} + P_{47}](1 - P_{24}P_{47})\}$

(49-50)

Now, the expected uptime of the system at epoch t is given by

$$\mu_{up}(t) = \sum_{x=0}^{t} A_0(x)$$

so that

$$\mu_{up}(h) = \frac{A_0(h)}{1-h}$$

8 Busy Period Analysis

8.1 Case-I

Let $B_i(t)$ be the probability of the inspector who inspect the kind of failure of a failed unit before being repaired by repairman. Using simple probabilistic arguments, as in case of reliability and availability analysis the following recurrence relations can be easily developed.

$B_0(t) = q_{01}(t-1)© B_1(t-1)$

$B_1(t) = Z_1(t) + q_{12}(t-1)© B_2(t-1) + q_{13}(t-1)© B_3(t-1)$

$B_2(t) = q_{20}(t-1)© B_0(t-1) + q_{21}(t-1)© B_1(t-1) + q_{24}(t-1)© B_4(t-1)$

$B_3(t) = q_{30}(t-1)© B_0(t-1) + q_{31}(t-1)© B_1(t-1) + q_{35}(t-1)© B_5(t-1)$

$B_4(t) = Z_4(t) + q_{41}(t-1)© B_1(t-1) + q_{42}(t-1)© B_2(t-1) + q_{43}(t-1)© B_3(t-1 + q_{46}(t-1)© B_6(t-1) + q_{47}(t-1)© B_7(t-1)$

$B_5(t) = Z_5(t) + q_{51}(t-1)© B_1(t-1) + q_{52}(t-1)© B_2(t-1) + q_{53}(t-1)© B_3(t-1) + q_{58}(t-1)© B_8(t-1) + q_{59}(t-1)© B_9(t-1)$

$B_6(t) = q_{62}(t-1)© B_2(t-1)$

$B_7(t) = q_{72}(t-1)© B_2(t-1)$

$B_8(t) = q_{82}(t-1)© B_2(t-1)$

$B_9(t) = q_{93}(t-1)© B_3(t-1)$

(51-60)

By taking geometric transformation and solving the equation

$$B_0(h) = \frac{N_3(h)}{D_2(h)}$$

The probability that the inspection facility is busy in inspecting the failure of failed unit is given by

$$B_0 = \lim_{t\to\infty} B_0(t)$$

Hence, by applying 'L' Hospital Rule, we get

$$B_0 = -\frac{N_3(1)}{D'_2(1)} \quad (61)$$

where

$N_3(1) = \mu_1\{1 - P_{24} + P_{24}P_{41}\}(1 - P_{24}P_{47}) + \mu_4 P_{24}(1 - P_{24}P_{47})$. (62)

and $D'_2(1)$ is the same as in availability analysis.

8.2 Case-II

Let $B'_i(t)$ be the probability that the repair facility is busy in repairing of failed unit when the system initially starts from regenerative state S_i. Using simple probabilistic arguments, the following recurrence relations can be easily developed.

$B'_0(t) = q_{01}(t-1)© B'_1(t-1)$

$B'_1(t) = q_{12}(t-1)© B'_2(t-1) + q_{13}(t-1)© B'_3(t-1)$

$B'_2(t) = Z_2(t) + q_{20}(t-1)© B'_0(t-1) + q_{21}(t-1)© B'_1(t-1) + q_{24}(t-1)© B'_4(t-1)$

$B'_3(t) = Z_3(t) + q_{30}(t-1)© B'_0(t-1) + q_{31}(t-1)© B'_1(t-1) + q_{35}(t-1)© B'_5(t-1)$

$B'_4(t) = Z_4(t) + q_{41}(t-1)© B'_1(t-1) + q_{42}(t-1)© B'_2(t-1) + q_{43}(t-1)© B'_3(t-1) + q_{46}(t-1)© B'_6(t-1) + q_{47}(t-1)© B'_7(t-1)$

$B'_5(t) = Z_5(t) + q_{51}(t-1)© B'_1(t-1) + q_{52}(t-1)© B'_2(t-1) + q_{53}(t-1)© B'_3(t-1) + q_{58}(t-1)© B'_8(t-1) + q_{59}(t-1)© B'_9(t-1)$

$B'_6(t) = Z_6(t) + q_{62}(t-1)© B'_2(t-1)$

$B'_7(t) = Z_7(t) + q_{72}(t-1)© B'_2(t-1)$

$B'_8(t) = Z_8(t) + q_{82}(t-1)© B'_2(t-1)$

$B'_9(t) = Z_9(t) + q_{93}(t-1)© B'_3(t-1)$

(63-72)

By taking geometric transformation and solving the equation

$$B'_0(h) = \frac{N_4(h)}{D_2(h)}$$

The probability that the repair facility is busy in repairing the failure of failed unit is given by

$$B'_0 = \lim_{t\to\infty} B'_0(t)$$

Hence, by applying 'L' Hospital Rule, we get

$$B_0^{'} = -\frac{N_4(1)}{D_2'(1)} \tag{73}$$

where

$N_4(1) = \mu_2 (1 - P_{24}P_{47}) + \mu_4 P_{24}(1 - P_{24}P_{47}) + \mu_6 P_{24} [P_{46} + P_{47}](1 - P_{24}P_{47})$ (74)

and $D_2'(1)$ is the same as in availability analysis.

9 Profit Function Analysis

The expected total profit in steady-state is

$$P = C_0A_0 - C_1 B_0 - C_2 B_0^{'} \tag{75}$$

where

C_0: be the per unit up time revenue by the system

C_1 & C_2: be the per unit down time expenditure on the system

10 Graphical Interpretation

The behaviour of the MTSF and the profit function w.r.t failure rate and repair rate have been studied through graphs by fixing the values of certain parameters a, b, C_0 , C_1 and C_2 as

a = 0.4, b = 0.6, C_0 = 400, C_1 = 100 and C_2 = 200.

On the basis of the numerical values taken as:

P = 122.9535, r = 0.15 and s = 0.85

The values of various measures of system effectiveness are obtained as:

Mean time to system failure (MTSF) = 15.50327.

Availability (A_0) = 0.885688.

Busy period of Inspector (B_0) = 0.17794.

Busy period of repairman ($B_0^{'}$) = 1.067639.

Figure: 2 show the behavior of MTSF w.r.t failure rate (p_2).It appears from graph that MTSF decreases with increase in failure rate.

Figure: 3 show the behavior of MTSF w.r.t repair rate (r). It appears from graph that MTSF increases with increase in repair rate.

Figure: 4 show the behavior of Profit function w.r.t failure rate (p_2) for different values of repair rate (r). It appears from graph that Profit decreases with increase in failure rate. Following observations have also been observed from the graph:

- For r = 0.3, profit function P > or = or < 0 as the failure rate p_2 < or = or >0.721. So the system is preferable only if the failure rate is less than 0.721.
- For r = 0.35, profit function P > or = or < 0 as the failure rate p_2 < or = or >0.811. So the system is preferable only if the failure rate is less than 0.811.
- For r = 0.4, profit function P > or = or < 0 as the failure rate p_2 < or = or > 0.954. So the system is preferable only if the failure rate is less than 0.954

Figure: 5 show the behavior of Profit function w.r.t repair rate (r) for different values offailure rate (p_2). It appears from graph that Profit increases with increase in repair rate. Following observations have also been observed from the graph:

- For p_2 = 0.3, profit function P > or = or < 0 as the repair rate r > or = or < 0.151. So the system is preferable only if the repair rate is greater than 0.151.
- For p_2 = 0.4, profit function P > or = or < 0 as the repair rate r > or = or < 0.125. So the system is preferable only if the repair rate is greater than 0.125.
- For p_2= 0.6, profit function P > or = or < 0 as the repair rate r > or = or < 0.0578. So the system is preferable only if the repair rate is greater than 0.0578.

11 Conclusion

This paper concluded that the preventive maintenance of units increases both the availability and profit of the system by providing the numerical results for MTSF, availability and busy period of repairman and inspector. It also provides information for other researchers and companies following such systems to prefer the equipments which satisfied the conditions as discussed.

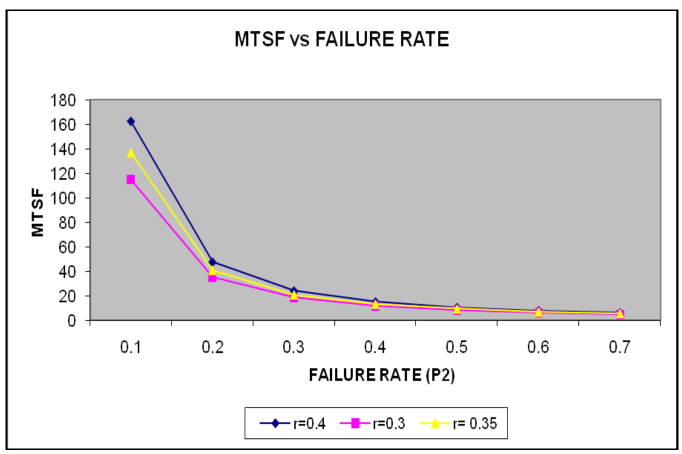

Figure: 2

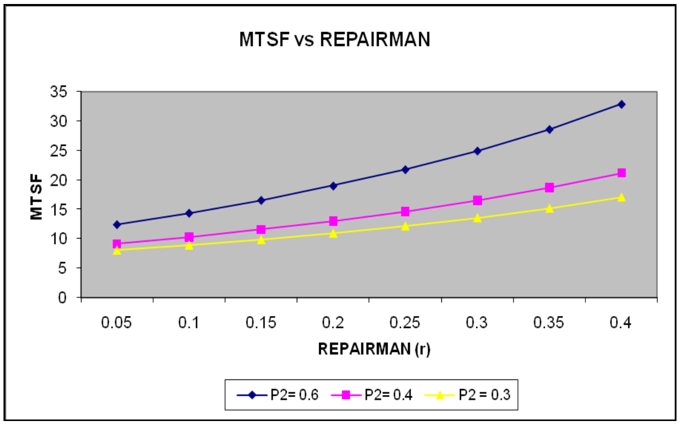

Figure: 3

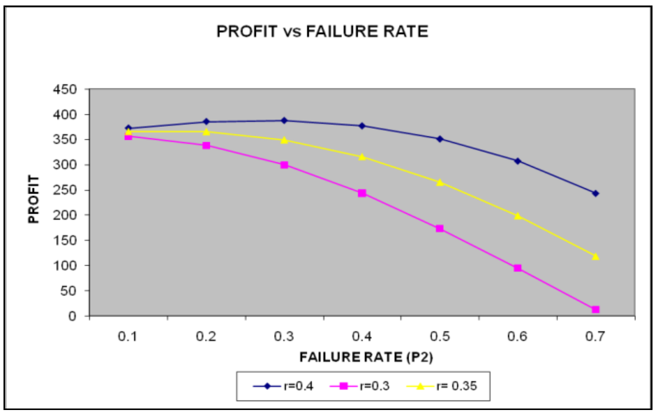

Figure: 4

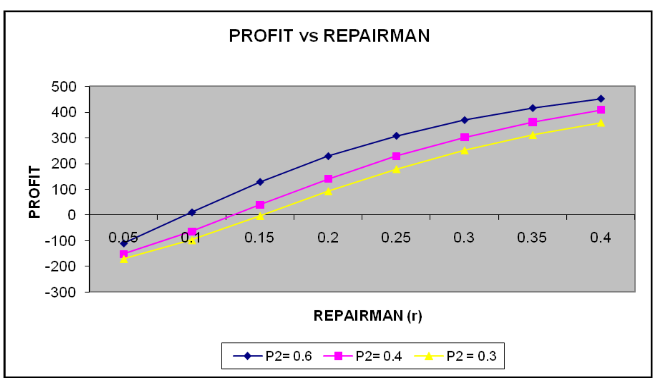

Figure: 5

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